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# A GEOMETRIC PICTURE OF THE FIFTEEN SCHOOL GIRL PROBLEM.

By PROF. ELLERY W. DAVIS, Lincoln, Neb.

The problem is to walk out 15 girls by threes, daily for a week, without ever having the same two together.

Instead of girls think of 15 points distributed 8 at the corners of a cube, 6 at the mid-points of the faces, one at the cube's centre.

From them form 35 triads no two having a pair of points in common, thus :—

The centre of the cube with each pair of opposite corners gives 4 triads. Call each triad an *a*.

The centre of the cube with each pair of mid-points of opposite faces gives 3 triads. Call each a *b*.

From the 24 triangles, 4 to each face, formed by drawing the face diagonals, select 12. Call each a *c*.

From the faces of the co-axial inscribed tetrahedron select 4. Call each a *d*.

Finally, each face-diagonal is a side of a triangle of which the mid-point of the opposite face is a vertex. There are 12 such. Call each an *e*.

We now easily form the seven sets of 5 triads.

With each *a* take the perpendicular *d* and the 3 *c* that have no points in common with that *a* or *d*. This gives 3 of the sets.

With each *b* take 4 *e*-that have no points in common with it or with each other. This can be done in two ways for the first *e* that is taken, but the way then chosen leaves no choice for the other 3 *e*. Thus we have the remaining 4 sets.

It but remains to establish a one-to-one correspondence between the points and the girls.

The system is not 7-cyclic. To prove this notice that if *x, y, z* is a triad on a face of the cube and *o* the cube's centre, while *x', y', z'* are opposite to *x, y, z* respectively ; then *xox', yoy', zoz'* and *x'y'z'* are all triads of the system. Were the system 7-cyclic such a set of 5 triads from 7 elements could not be formed. Try to do so.

The three types of 7-cyclic systems have for Sunday groupings, adopting a common notation,

- |    |  |
|----|--|
| A. | $ka_1b_1, \quad a_2a_3a_5, \quad a_4b_5b_7, \quad a_6b_3b_4, \quad a_7b_2b_6;$ |
| B. | $ka_1b_1, \quad a_2a_3a_5, \quad a_7b_5b_4, \quad a_4b_3b_6, \quad a_6b_2b_7;$ |
| C. | $ka_1b_1, \quad a_2a_3a_5, \quad a_4b_2b_6, \quad a_6b_5b_7, \quad a_7b_3b_4.$ |

For  $o$  in the  $xyzox'y'z'$  set  $k$  cannot be taken ; for then,  $aaa$  for  $xyz$  would require  $bbb$  for  $x'y'z'$ , while  $abb$  would require  $baa$ .

Neither can we have a  $b$  for  $o$ . This would require with  $kab, aaa, abb$  respectively  $aba, bbb$  or  $bbk, baa$ .

Finally  $a$  wont do for  $o$ . Here the indices must be considered. The index triads are

$$-, i, i; \quad i, i + 1, i + 3; \text{ and } i, i + s, i - s;$$

the last form occurring only in  $C$ . Then,  $j$  is the index of the  $a$  chosen for  $o$  ;

$-, i, i$  would require  $j, l, l$  in  $A$  or  $B$  and  $j, l, 2j - i$  in  $C$ , the indices belonging to different letters.

$i, i + 1, i + 3$  would require (in some order)  $l, l - 1, l - 3$  in either  $A, B$ , or  $C$ .

$i, i + s, i - s$  would require (of course in  $C$ )  $l, 2j - i - s, 2j - i + s$  with  $l \neq 2j - 1$ .

Each and every requirement being impossible to fulfil the system we have imagined on the cube the system is proved non 7-cyclic.